Day 13

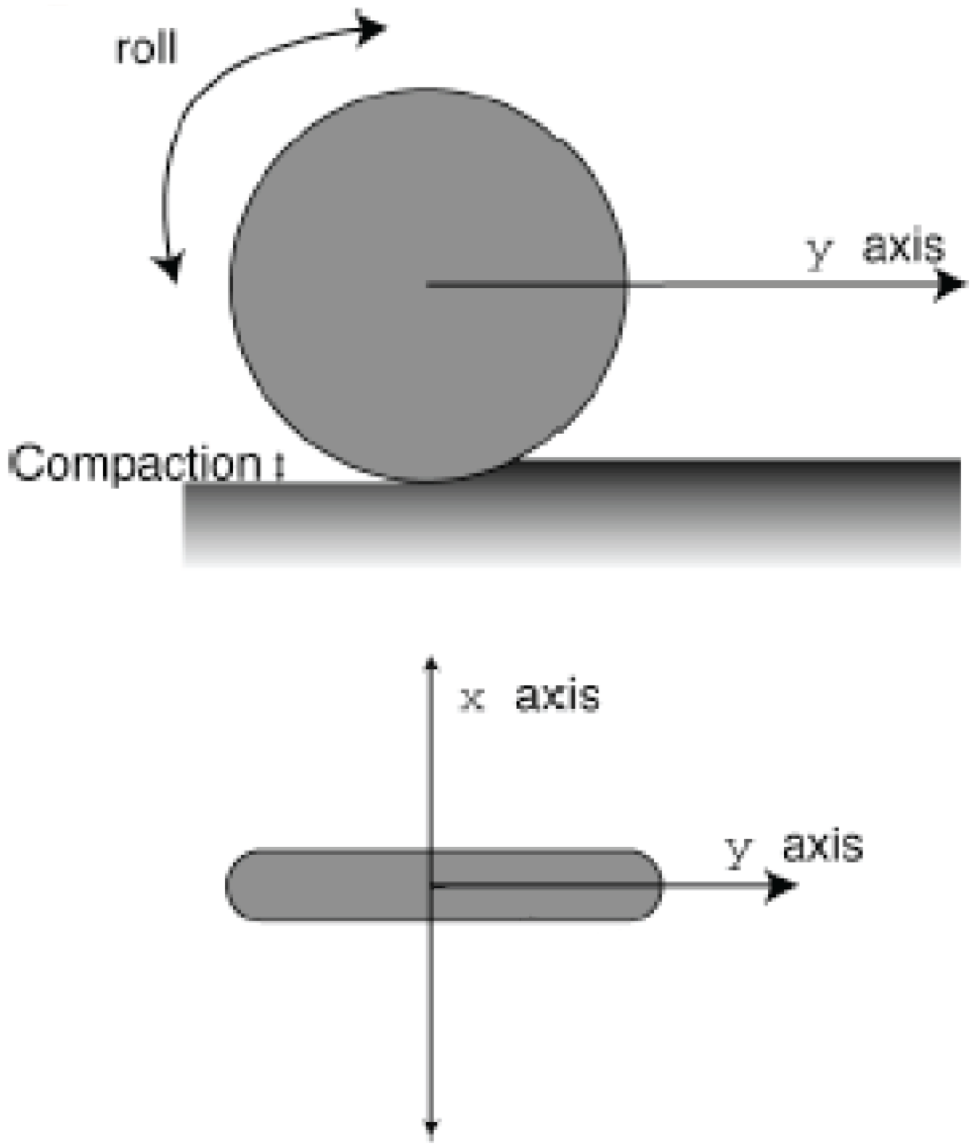
Kinematics of Wheeled Robots

- https://www.youtube.com/watch?v=giS41utjlbU

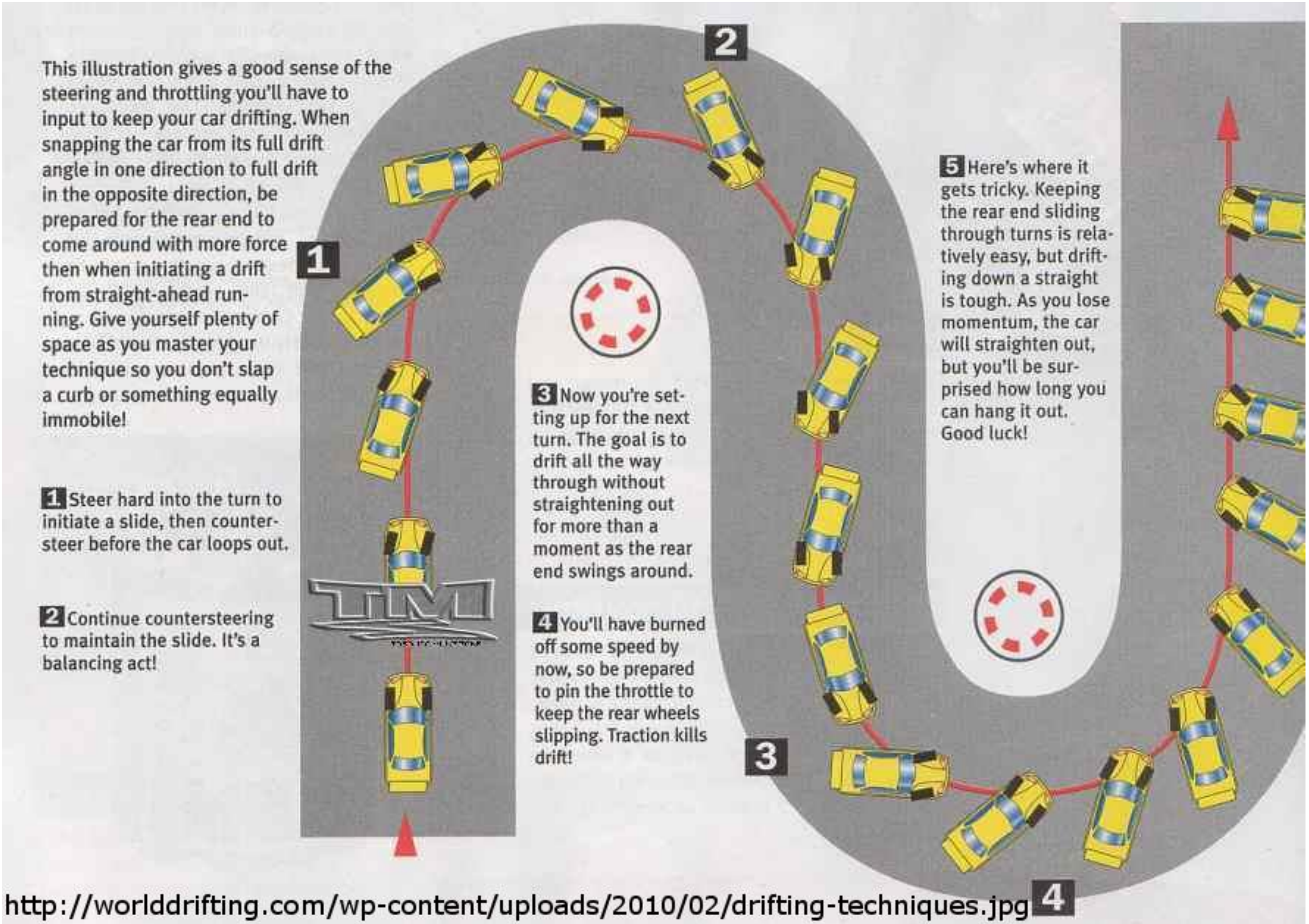
Wheeled Mobile Robots

- robot can have one or more wheels that can provide
  - steering (directional control)
  - power (exert a force against the ground)
- an ideal wheel is
  - perfectly round (perimeter $2\pi r$)
  - moves in the direction perpendicular to its axis

Wheel



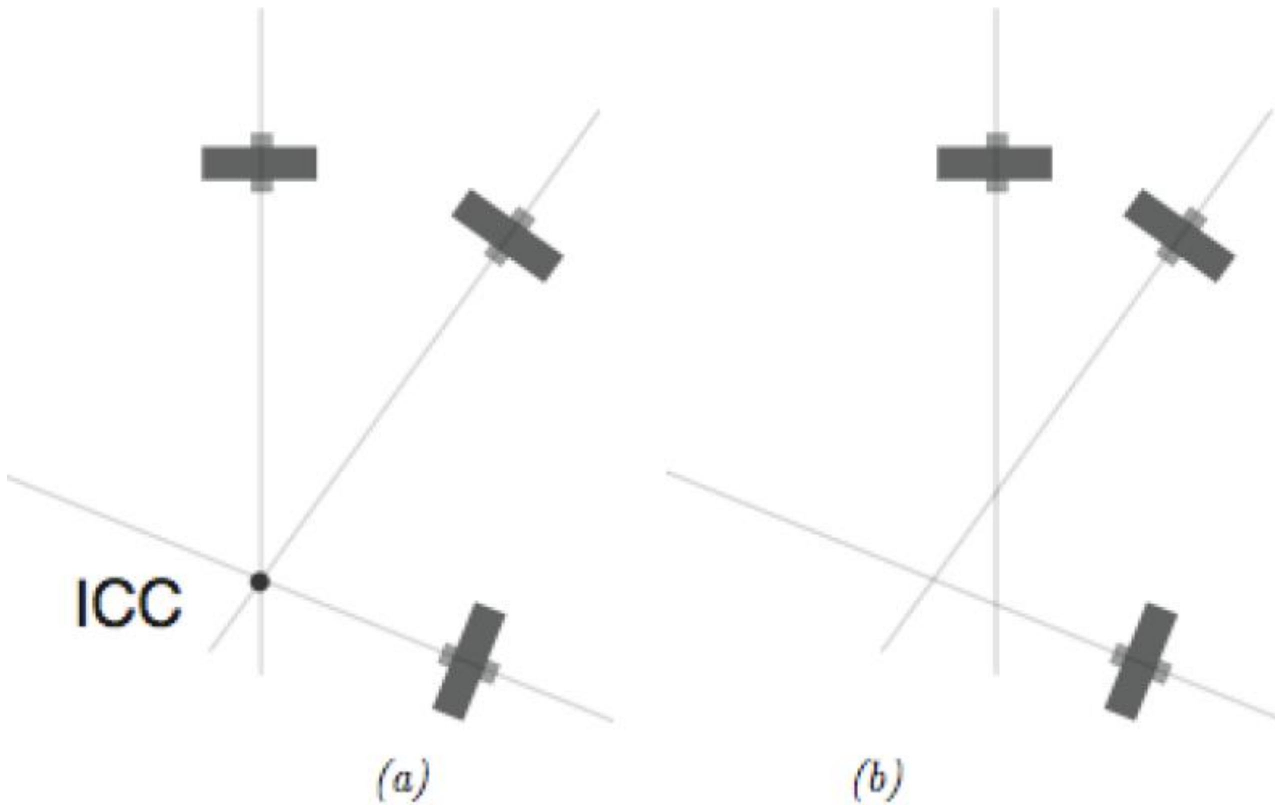
Deviations from Ideal



Instantaneous Center of Curvature

- for smooth rolling motion, all wheels in ground contact must
  - follow a circular path about a common axis of revolution
    - each wheel must be pointing in its correct direction
  - revolve with an angular velocity consistent with the motion of the robot
    - each wheel must revolve at its correct speed

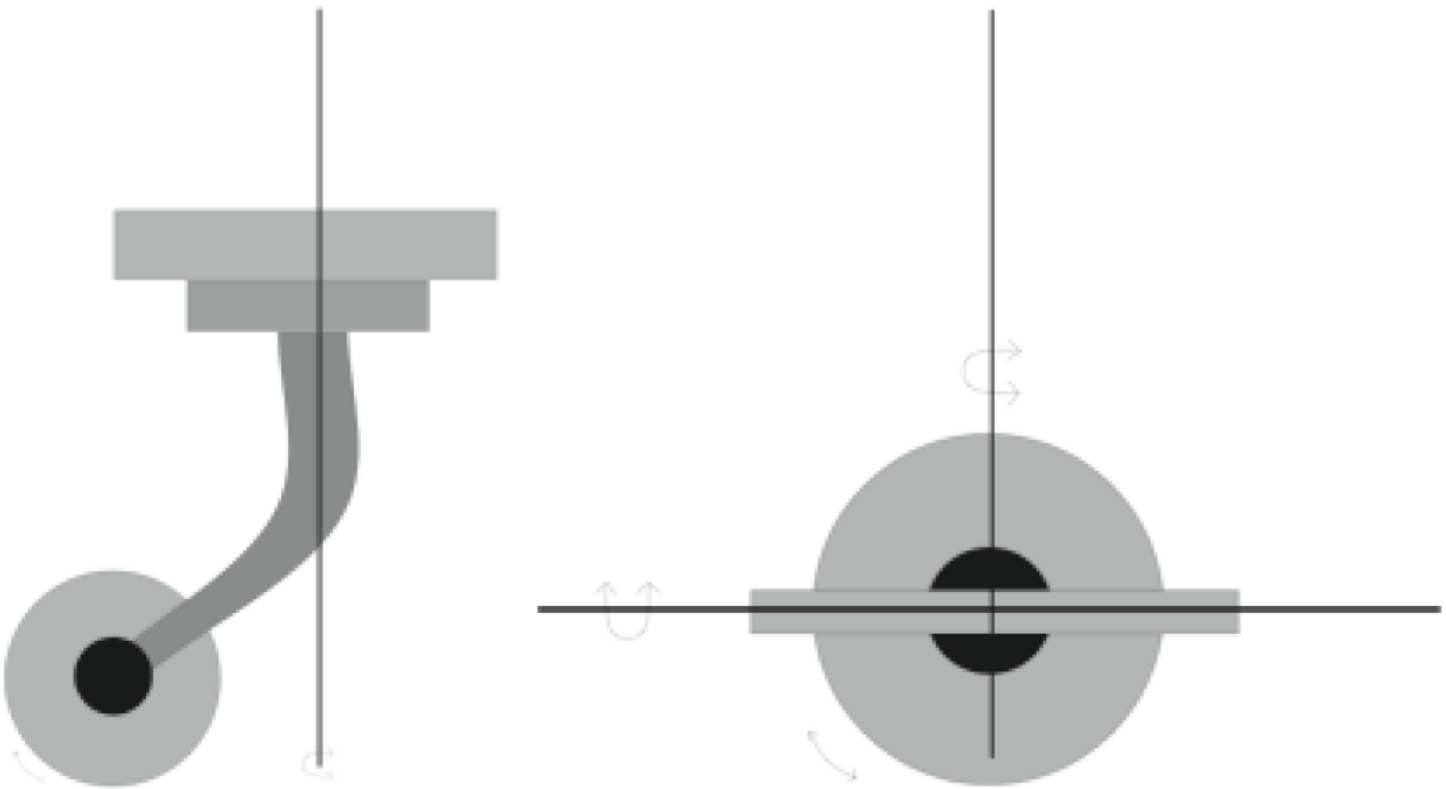
Instantaneous Center of Curvature



(a) 3 wheels with roll axes intersecting at a common point (the instantaneous center of curvature, ICC). (b) No ICC exists. A robot having wheels shown in (a) can exhibit smooth rolling motion, whereas a robot with wheel arrangement (b) cannot.

Castor Wheels

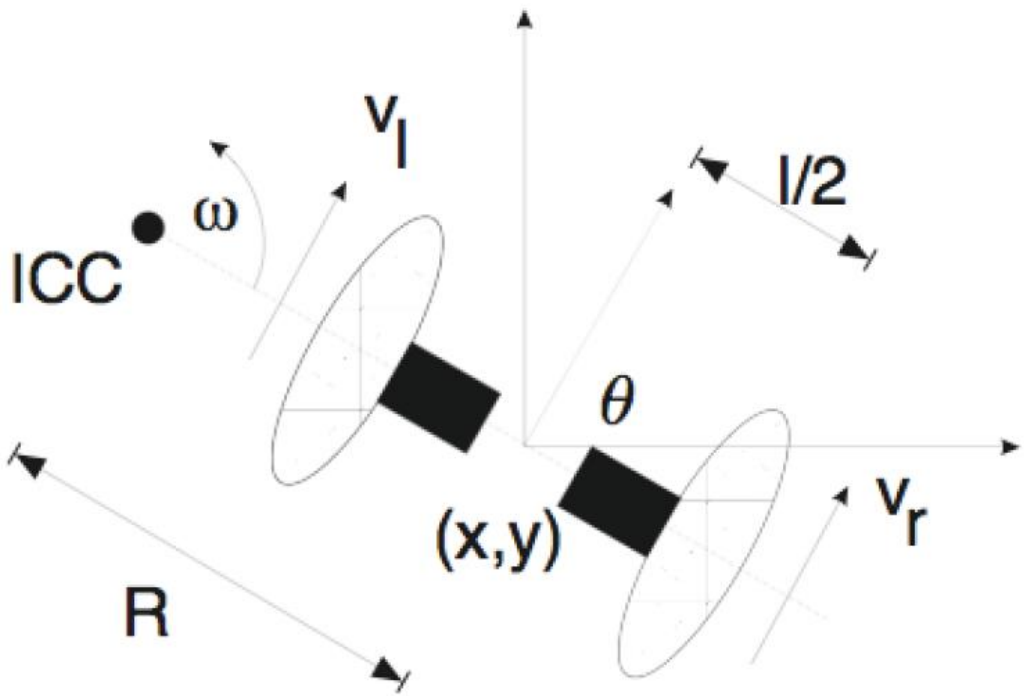
- provide support but not steering nor propulsion



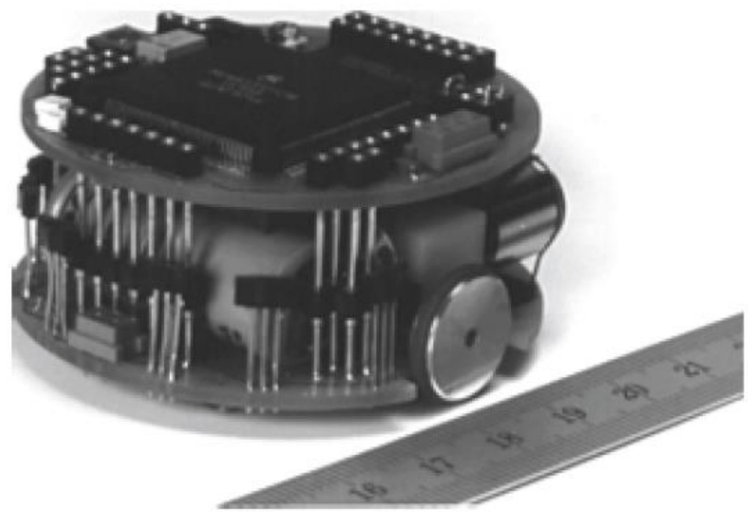
(a) Castor Wheel (b) Rollerball wheel

Differential Drive

- two independently driven wheels mounted on a common axis



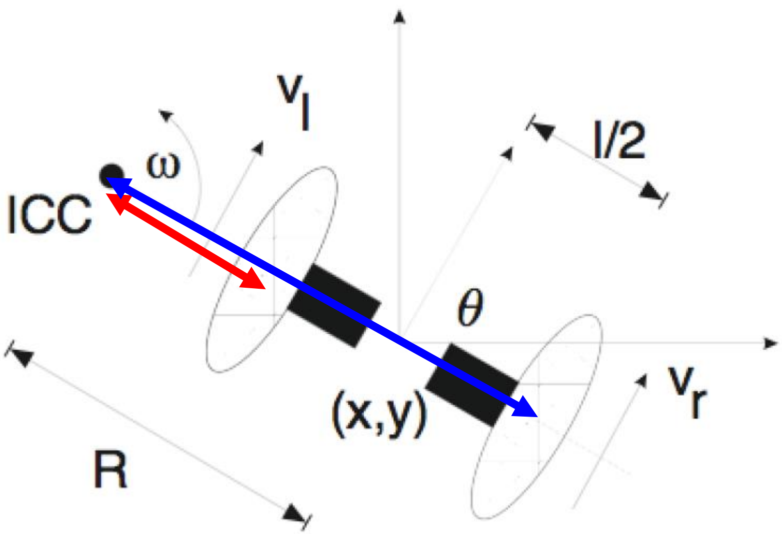
(a) Differential Drive



(b) Khepera Robot

Differential Drive

- angular velocity ω about the ICC defines the wheel ground velocities v_r and v_ℓ



(a) Differential Drive

$$v_r = \omega(\overbrace{R + \frac{\ell}{2}})$$

$$v_\ell = \omega(\underbrace{R - \frac{\ell}{2}})$$

Differential Drive

- given the wheel ground velocities it is easy to solve for the radius, R, and angular velocity ω

$$R = \frac{\ell}{2}\frac{\left(v_r + v_\ell\right)}{\left(v_r - v_\ell\right)}$$

$$\omega = \frac{\left(v_r - v_\ell\right)}{\ell}$$

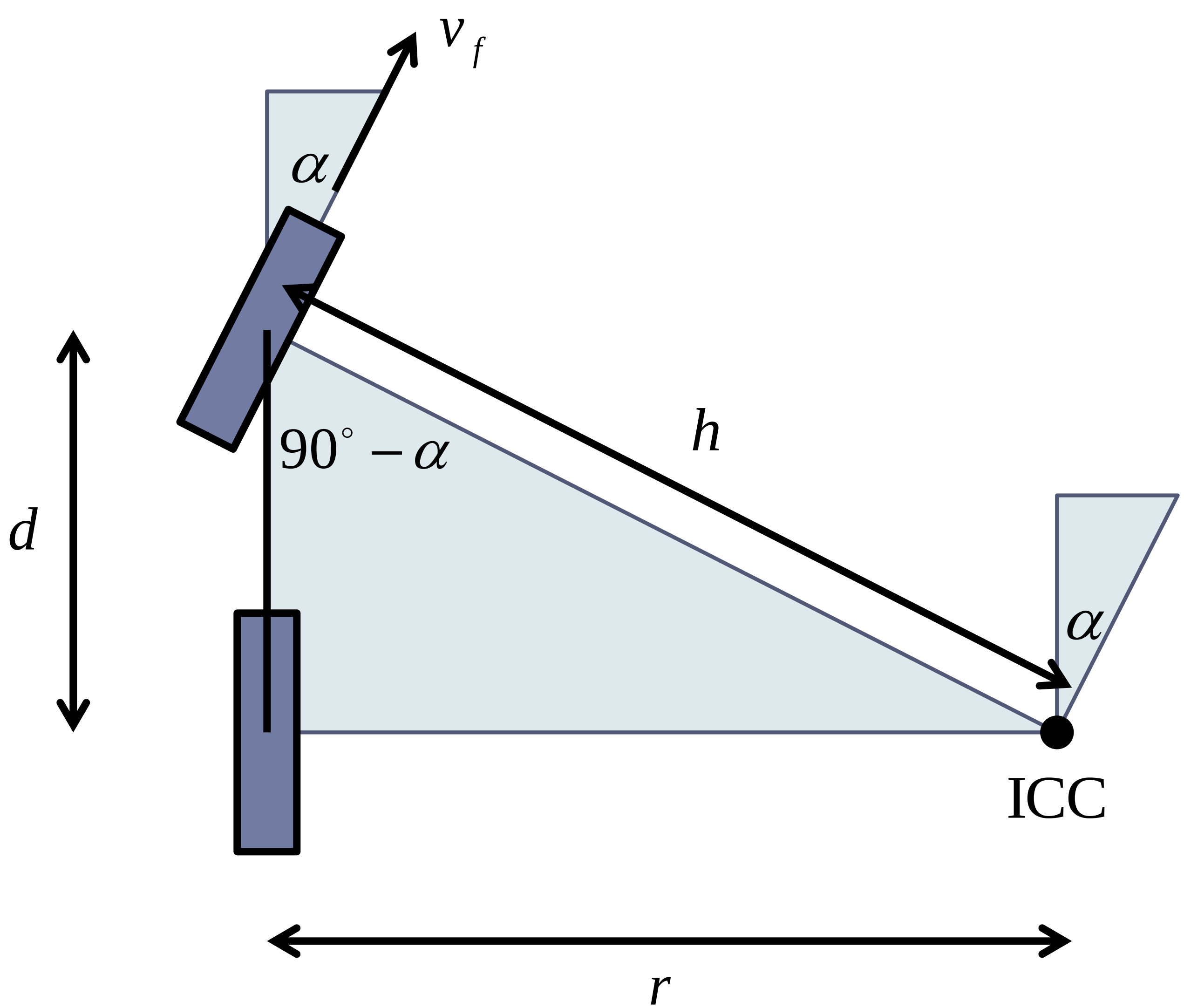
- interesting cases:
  - $v_\ell = v_r$
  - $v_\ell = -v_r$

Tracked Vehicles

- similar to differential drive but relies on ground slip or skid to change direction
  - kinematics poorly determined by motion of treads

http://en.wikipedia.org/wiki/File:Tucker-Kitten-Variants.jpg

Steered Wheels: Bicycle



Steered Wheels: Bicycle

- important to remember the assumptions in the kinematic model
  - smooth rolling motion in the plane
- does not capture all possible motions
  - http://www.youtube.com/watch?v=Cj6ho1-G6tw&NR=1#t=0m25s

Mecanum Wheel

- a normal wheel with rollers mounted on the circumference

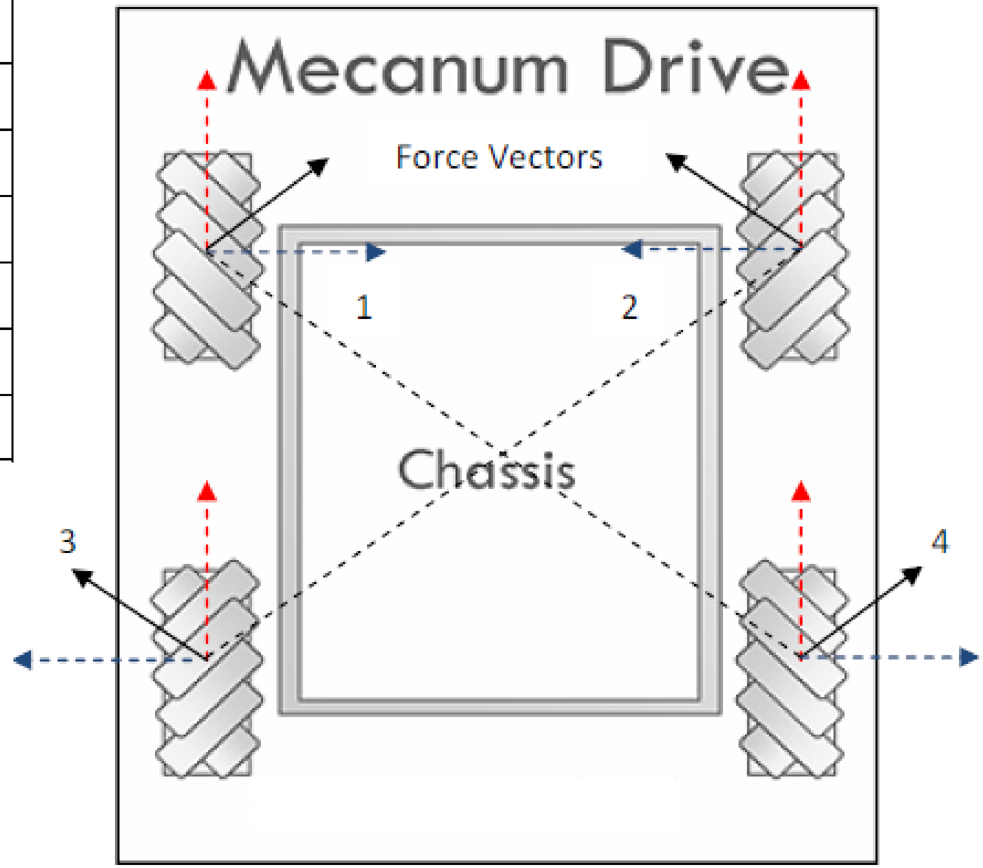
http://blog.makezine.com/archive/2010/04/3d-printable-mecanum-wheel.html

- http://www.youtube.com/watch?v=Cee1UZN0p98&feature=player_embedded

Mecanum Wheel

Direction of Movement	Wheel Actuation
Forward	All wheels forward same speed
Reverse	All wheels backward same speed
Right Shift	Wheels 1, 4 forward; 2, 3 backward
Left Shift	Wheels 2, 3 forward; 1, 4 backward
CW Turn	Wheels 1, 3 forward; 2, 4 backward
CCW Turn	Wheels 2, 4 forward; 1, 3 backward

To the right: This is a top view looking down on the drive platform. Wheels in Positions 1, 4 should make X- pattern with Wheels 2, 3. If not set up like shown, wheels will not operate correctly.



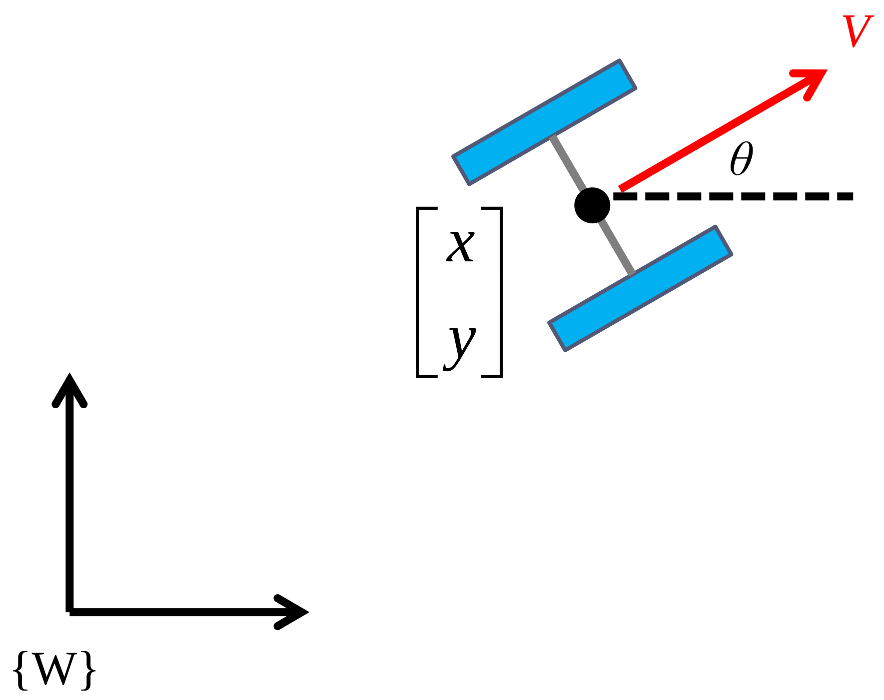
AndyMark Mecanum wheel specification sheet
http://d1pytrrjwm20z9.cloudfront.net/MecanumWheelSpecSheet.pdf

Forward Kinematics

- serial manipulators
  - given the joint variables, find the pose of the end-effector
- mobile robot
  - given the control variables as a *function of time*, find the pose of the robot
    - for the differential drive the control variables are often taken to be the ground velocities of the left and right wheels
      - it is important to note that the wheel velocities are needed as functions of time; a differential drive that moves forward and then turns right ends up in a very different position than one that turns right then moves forward!

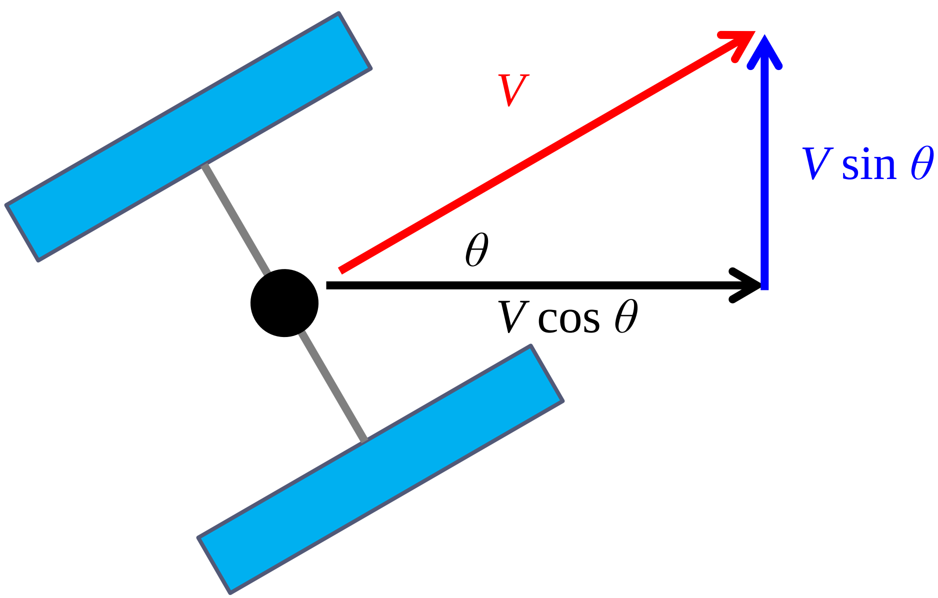
Forward Kinematics

- robot with pose $[x \;\; y \;\; \theta]^T$ moving with velocity V in a direction θ measured relative the x axis of $\{W\}$:



Forward Kinematics

- for a robot starting with pose $[x_0 \;\; y_0 \; \theta_0]^T$ moving with velocity $V(t)$ in a direction $\theta(t)$:



$$x(t) = x_0 + \int_0^t V(t)\cos(\theta(t))\,dt$$

$$y(t) = y_0 + \int_0^t V(t)\sin(\theta(t))\,dt$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t)\,dt$$

Forward Kinematics

- for differential drive:

$$x(t) = \frac{1}{2}\int_0^t (v_r(t) + v_\ell(t))\cos(\theta(t))\,dt$$

$$y(t) = \frac{1}{2}\int_0^t (v_r(t) + v_\ell(t))\sin(\theta(t))\,dt$$

$$\theta(t) = \frac{1}{\ell}\int_0^t (v_r(t) - v_\ell(t))\,dt$$

Sensitivity to Wheel Velocity

$$v_r(t) = 1 + \mathcal{N}(0, \sigma^2)$$

$$v_\ell(t) = 1 + \mathcal{N}(0, \sigma^2)$$

$$\theta(0) = 0$$

$$t = 0 \ldots 10$$

$$\ell = 0.2$$